Options Embedded in Physical Money

Working Paper Short Version

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9 April 2009

Abstract

This article introduces the concept of embedded options in the world's physical monies, both coin and paper. The option value for base metal coins is presented. The various strategies for redemption by the owner and the prevention of redemption by the issuer (central banks) are discussed. The market values of gold coins are discussed in light of the embedded option valuation. In conclusion, the rational behavior of both individuals and central banks in light of these valuations is described.

Key Words: Coins, bills, paper money, central bank, monetary policy and regulation, embedded American perpetual put options, knock-in perpetual options, melting of coins.

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1 Introduction

The American put options embedded in most coin currencies, as well as paper notes, are options poorly understood by the issuer or holder, almost undescribed in the literature, and rarely considered with options in view. Even less frequently recognized is a special barrier feature associated with these options. Since the history of currencies most likely begins with the use of the underlying value of the metal in the coin, the lack of research is surprising. The shift of most money to electronic representation greatly increases the importance of these options. This lack of attention is additionally surprising since the loss of the ability to redeem a paper US dollar for its underlying value in silver (Silver Certificate) occurred within the last century. In fact, the value of modern currencies has recently gone from deep in-the-money through at-the-money to again deep-in-the-money, making the lack of sound analytical treatment all the more surprising.

Beck and Stockman (2005) is the only paper we are aware of that tries to connect money to option theory. The authors consider money as a perpetual American call option on goods. We do not disagree with this. What we point out in this article are some very different embedded options found in relation to physical and non-physical money. With physical money, we think of money made of metal or even paper, and non-physical money are electronic money.

The study of the embedded options in physical money can bear fruit in a number of ways. Option theory provides new insights into how people actually handle money, why people usually keep their money in the bank, and at other times withdraw large parts of their money from the bank to store them or consider melting or even burning them. In addition, knowledge of the fact that the physical objects that the vast majority of people handle daily contain options may help demystify the option and derivatives markets. Finally, a better understanding of the option elements embedded in physical money could change both central bank policies and peoples' behavior towards money in certain situations.

One of the sovereign functions of a country is the issuance of physical money and the definition of the rules for redemption. In today's world, most money is actually represented electronically. Electronic money can, however, be exchanged at any time into physical money. The impact of the rational exercise of the embedded option causing the movement of money between its physical and electronic forms is one of the objectives of this article.

The value of metal used in coins can vary dramatically over time, and this variation can naturally affect the value of the coin. Coins also have a value in the form of the face value printed on them. A coin is a piece of (long) metal that has a minimum price guaranteed by the government. The face value of a coin can be seen as the strike price. At any time, the owner of the coin (a piece of metal) has the right to exchange it for materials or services worth the face value printed on the coin. Thus, the holder of the coin also owns the option to exercise for the face value. One way to exercise the option is to deliver the physical money to a commercial bank as a bank deposit. Money in the bank generates interest, but the owner no longer possesses the physical coins. To hold money physically, therefore, has a negative cost of carry equal to the interest a bank would pay. In addition, the secure storage cost of the coins may be important.

If there is an assumption that the government cannot default on its own currency, then a coin can typically be seen as a unit of long base metal with an embedded American put option with infinite time to maturity, better known as a perpetual option. In the case of credit risk, the expected maturity can naturally be considerably shorter. For example, during a war or revolution, the existing money could stop being legal tender. Of course, if one form of money can be exchanged into other forms (e.g., electronic or paper money into coins at the face value of the coin), then if the coin is incorrectly priced an arbitrage opportunity may exist. If the base metal value of the coin is significantly higher than the face value, an arbitrageur will then not necessarily care if the coin plus the embedded put option should have an even higher theoretical value. In this case, the option value will typically also be very small, and it will still cost to hold the coin (losing interest rate). The arbitrageur will simply melt the coin and sell it for its base metal value. Throughout history, circulated coins have from time to time been priced at less than their base metal value. Central banks and their governments typically hold a fixed price for their base metal coins, while for precious metal coins the governments tend to adjust the price in line with the value. This behavioral difference will inform the option valuation analysis below.

Historically, the price of the metal in most base metal coins is far below the face value. Such put options are, therefore, deep-in-the-money. The value of most base metal coins for this reason is almost exactly the face value of the coin. The ability to exchange bank deposits (electronic money) into physical money is limited by the availability of the physical money. Conversely, the physical money will only be widely available if the metal plus the embedded option has a value close to the face value. As always, arbitrageurs ensure that these relationships exist over time. Historically, there have been potential arbitrage opportunities that lasted for years.

Economists already have some current models of when the public will start to melt coins (see Sarget and Velde (2002)). These existing models do not take into account the embedded option aspect of the money. The more traditional models of coins assume the public will start to melt the coins when the metal value is above the face value of the coin, in terms of other goods. In recent times, it looks like people have waited considerably longer before they start melting coins, something that is more consistent with the assumption that coins typically contain embedded options.

If the metal value exceeds the face value, melting and selling at the minimum metal value redeem the underlying value. Only if the buyer of the metal actually needs to use the metal is melting necessary. Often, metal is simply used as an investment and simply stored in its current form. The coins then continue to have the metal value plus the option value. For this reason, one could be led to think it would never be optimal to melt the coin, as the coin should always have the metal value plus the option value, which should exceed the metal value. It is, however, more complicated than this. In practice, everyone has a limited amount of capital. To buy coins will bind up your capital. When the metal value is considerably above the face value, the option value will at the same time be very small. So, in this case, you hardly have any option value, but it still cost to store the coins — that is, to not have your money in a bank account or in other interest bearing securities. By storing coins, you lose interest income and also lose out on alternative use of your resources. When the metal value is considerably above the face, it is therefore probably optimal at some point to melt the coin and sell the metal — in particular because, after selling the metal of the melted coins, you can potentially go back to the bank and buy more of the same coins for the price face value of the coin. However, as we will see, it is not always possible to go back and buy more of the same coins.

Exactly when someone will start melting the coins is hard to model, but not impossible, and by taking into account the embedded options in coins we are at least getting closer to understanding when it is optimal to melt coins. This is somewhat parallel to American call options on non-dividend-paying stocks. Here, academics in their famous models actually assume it never will be optimal to exercise the option before expiration. Still, we see practitioners quite often exercising such options before expiration. Can part of the explanation be because people in the market act irrationally?[1] The more likely reason is because every investor has a limited amount of capital available. When an option is deep-in-the-money, it binds up a lot of capital. To get access to this capital now, you either have to sell the option or exercise the option. Others also have limited amounts of capital and would be inclined not to invest in deep-in-the-money options except at a good price, in particular when there is lack of liquidity. This means that sometimes they would not even offer the instinct value of the option, and in this case you are typically better off by simply exercising the option if you need to free up capital. So, yes, even if academics claim that it is not optimal to exercise American options before expiration, this is often optimal in practice. For coins, this example is in many ways turned upside down, as the option is embedded in the underlying asset; the option cannot easily be separated from the coin itself. It is when the embedded option is deep out-of-the-money that you are binding up maximum capital relative to what you get. The coin still costs the face value, and when the metal value is considerably above the face value, the option value is close to zero. For the increase in the level of the metal price above the face value in order for the option to have close to zero value will, of course, again depend on the volatility level and several other factors that we will look at more closely below. In this case, it could be optimal to melt the coin even if you lose the option. The option value you lose is very small, and you will free up capital and save storage costs. Most importantly, if the coins keep trading at face value, you can go back and buy more coins and arbitrage the market. In a world with unlimited capital, and if the coins were always priced to reflect the option value, it would not be rational to melt such coins, but, again, in the real world there is a limited amount of capital, and

[1] Even if this conflict with the same models and academics basically assuming efficient markets and full rationality.

also limited amount of metals and also at least base metal coins tend to trade at face value, even when the metal value exceeds the face value, at least for some period of time. We will not try to come up with an exact mathematical model for the point where people should melt their coins; we will leave this up to others. However, the fact that coins contain an embedded option and that the option has a high time value around at-the-money can explain why people in recent times have not been rushing to melt coins, even if the metal value has been above the face value of the coin. By melting the coin at that point, they would have lost the option value. When the metal value of US nickel (5 cent base metal coin) during most of 2007 was above the face value, people did not rush to melt coins. There were probably several reasons for this — one being that many people simply did not follow the base metal market; another reason was transaction costs relating to melting coins, but also the fact that the time value of the option will be relatively high as long as the metal value is not too far above the face value. Another reason was probably that many considered the melting of coins to be illegal. However, this was probably not the case in the USA back in 2006 and most of 2007, when several base metal coins had value far exceeding their face value. We think it is high time for economists and people to start recognizing the embedded options physical money. We will now look at the mathematical modeling of physical money, account the embedded optionality.

2 Valuation Theory

Consider that the face value of the coin is equal to the put strike price, X. The intrinsic value of the embedded option can be written as $\max[X - S, 0]$ where S is the physical value of the money (e.g., metal). The value of the metal plus the exercise value of the option is:

$$S + \max[X - S, 0] \tag{1}$$

This value is identical to the face value of the coin when the embedded option is in- or at-the-money. When the metal value of the coin is much lower than the strike price (the face value), the time value of the option is naturally also very low. Historically, most of the time well-circulated coins are deep-in-the-money and people are buying and selling them from a bank simply at their face value. Deep-in-the-money options are very easy to value, as almost all the value is simply the intrinsic value plus the metal value (little or no time value in the option).

The embedded option in coins is a American style option. Exercising a coin is somewhat different than for other options. For example, when a American put option on a stock is exercised, the stock is forfeited for money. In the case of a coin, the exercise forfeits the money and redeems electronic money. Possession of physical coins incurs two types of cost of carry: safe storage and the loss of interest that a bank would pay. When the option is deep-in-the-money and there is close to no time value in the embedded option, it is optimal to exercise the option. In today's society, exercising an embedded perpetual put coin option would typically mean delivering coins to the bank and exchanging them for electronic money.

For paper money, the printed face value on the paper money (e.g., a $20 bill) is the strike price of the embedded perpetual put option. Since paper is so cheap to produce, it is very unlikely that the value of the paper in a bill at any time in the foreseeable future will be anything close to the face value of the bill (except potentially in Zimbabwe were there currently is extreme hyper inflation.) In other words, all the embedded options in paper money are very deep in-the-money and have basically zero time value. It is therefore optimal to exercise the embedded option in paper currency into electronic money (or, in some situations, coins). It is interesting to note, however, that during the inflation period in Germany from 1923—1924, the face value of the paper money at some point was probably lower than that of the physical paper itself. At the peak of this time of hyperinflation, burning money was less expensive than buying firewood — that is, the embedded option in paper money actually went out-of-the money.

When physical money is exercised into electronic money, there is no guarantee that withdrawing the electronic money will recover the same physical money and embedded option. The withdrawal will be for legal tender with the same aggregate face value as the money deposited. If the government in the meantime excluded the coins that had been deposited, such coins could not be withdrawn, only deposited. Historically, this typically happens when the embedded option moves from deep-in-the-money to at- or even out-of-the-money. The value of a coin plus its option only has value equal to price when the option is deep-in-the-money, as is normally the case. If an embedded coin option moves towards at-the-money or out-of- the-money, and the central bank and the commercial banks are still selling such coins at the face value, as they normally do, it opens up arbitrage opportunities. The government will typically try to close down such opportunities over time by trying to regulate the value of the coin down to the price of the coin, where the price of the coin typically is equal to the face value of the coin.

It is far from certain that base metal coins have their option deep-in-the-money. With surging base metal prices over the past few years, more and more well-circulating coins have moved towards at-the-money, and some even towards deep-out-of-the-money. An example of such a coin is the 5 cent nickel coin in the USA. This coin, which is still widely circulating at the time of writing, was produced from 1946—2007 and is known as the Jefferson nickel. Each coin contains 0.0083 pounds of copper and 0.0028 pounds of nickel. With a copper price of $3.4632 per pound and nickel price of $22.3678 per pound (April 12, 2007), these commodity prices value one nickel coin at $3.4632 \times 0.0083 + 22.3678 \times 0.0028 = 0.09$. Thus, a 5-cent coin has a metal value of 9 cents and the embedded option is deeply out-of-the-money. Since 1946, the US government has minted over 50 billion Jeffererson nickels (source: Wikipedia). The government is naturally not happy with someone arbitraging the system, and they have several ways to put an end to such arbitrage opportunities. In 2006, there was more and more focus in the press about this arbitrage opportunity in base metal coins, and also indications that some people actually took advantage of it. The US Mint came out with a statement on December 14, 2006 making US coin melting illegal. The regulation also limits the export of nickels and pennies; a person can take a maximum of $5 of these coins in face value out of the country. The new regulations authorize a fine of not more than $10,000, or imprisonment of not more than five years, or both, against a person who knowingly violates the regulations. See Appendix A for the exact statement made by the United States Mint. In other words, the government has the opportunity to try to knock out the metal value of the coin. Based on this regulation, from the perspective of the coin holder, the coin can potentially be seen as a long perpetual put plus a piece of base metal, but at the same time you are short a knock-in perpetual call option on the metal value.

If the price of nickel increases much more, there is a big chance that the central bank at some point will declare the current nickel coin non-tender currency or replace it with a coin with a different and less valuable metal content. In 1960, the value of the silver in a silver dollar coin was $1.29. The number of Silver Certificate notes declined, since, when a note was redeemed for silver dollar coins, the note was shredded as the silver was no longer in the Treasury. On June 4, 1963, the production of Silver Certificates was abolished by Congress. In 1964, the redemption for silver dollars ended, and in 1968, the redemption for silver bullion ended (source: Wikipedia).

When coins are deposited when the option is deep-in-the-money, this represents exercising the put option. Then, after a year or two, assuming that the metal price has gone up dramatically, so that the metal value of the coins is worth four times that of the value of the coin itself, withdrawal of the deposited coins may not then be possible. The central bank has stopped issuing these particular coins and the local bank only accepts deposits of this type of coin. The withdrawal will only be made in newly issued coins. The newly issued coins could have the same face value, but they would certainly contain much less valuable metals. For example, the central bank could switch to a cheaper base metal or mix of metals and/or reduce the physical size of the coin. This removal from circulation has happened many times, in many different countries.

Ignoring the fact that governments have the possibility of making melting illegal, as well as taking coins out of circulation, the option embedded in the coin is a pure American "perpetual" put option. To value the embedded coin options, we need some idea about the volatility of the metal price. Base metals and precious metals are actively traded on several exchanges — the London Metal Exchange and NYMEX, and large quantities are also traded in the interbank market, where many large banks operate as market makers. So, we can get a good idea of the expected volatility, both from looking at the historical perspectives and also potentially at implied volatilities from exchange-traded options. By assuming that the metal price is following geometric Brownian motion:

$$dS = \mu S dt + \sigma S dz,$$

where μ is the expected instantaneous rate of return on the underlying asset, σ is the instantaneous volatility of the rate of return, and dz is a Wiener process.

Mckean (1965) studied the pricing of perpetual warrants in a non-risk-neutral world. Merton (1973) gives the formula for an American perpetual put option in a risk-neutral framework (based on a series of theoretical assumptions) by modifying Mckean (1965) technique, see also Gerber and Shiu (1994) and Haug (2007a):

$$p = \frac{X}{1-y}\left(\frac{y-1}{y}\frac{S}{X}\right)^{y}, \tag{2}$$

where X is the strike price and

$$y = \frac{1}{2} - \frac{b}{\sigma^2} + \sqrt{\left(\frac{b}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}.$$

where r is the risk-free rate and b is the cost of carry the underlying assets.

There is however plenty of evidence that commodity and asset price returns are non-Gaussian distributed. In practice, fat-tails and high-peaks in the price returns relatively to the theoretical Gaussian distributions are observed. That price data are not normally distributed dates at least back to Mitchell (1915), Oliver (1926) and Mills (1927). One reason for this is that jumps are observed in asset and commodity prices. See Aase (2005) for description of valuing perpetual American options under jump-diffusion. Delta hedging as an argument for risk-neutral valuation works particular poorly when there are jumps in the underlying asset. With jumps in the asset price, Carr and Wu (2002) shows how hedging options with options is superior to delta hedging, see also Derman and Taleb (2005), Haug (2007b), Haug and Taleb (2008) and Hyungsok and Wilmott (2008) for a detailed analysis and discussion on this topic. According to Carr and Wu simulations indicate that the inferior performance of the delta hedge in the presence of jumps cannot be improved upon by increasing the rebalancing frequency, see also Hyungsok and Wilmott (2007). Option traders to a large degree base their valuation on more robust hedging principles

As mentioned, if the metal prices go too high relative to the face value, the central bank in most countries will sooner or later step in and remove the arbitrage opportunity. The reason for this arbitrage opportunity is that the government does not sell most coins at the market value, but at a fixed price equal to the face value of the coin. They therefore have to use other means to bring the value down to their “artificially” fixed price. As mentioned earlier, they will take the coin out of circulation or make melting and exporting of the coin illegal. In the case of the coin being taken out of circulation, the government will typically provide a window period during which the coin can still be used as legal tender. In such a window period, you will be able to deposit such coins, but no longer withdraw (buy) them. The embedded perpetual American put option in the coin can then be seen as an option where the time to maturity is reset or knocked out to a much shorter time to maturity; see Haug and Haug (2001) for flexible ways to implement options when the time to maturity is reset. The resetting of the time to expiration on the option is typically a partly deterministic and partly stochastic function of the metal price. The higher the metal price trades relative to the face value, the more likely it is that the government will take the coin out of circulation and replace it with a coin with less valuable metal content. If the government makes melting and exporting of their coins illegal, the holder of the coin is short a American knock-in call on the metal value in addition to his long American perpetual put. The call will typically have a strike price equal to the face value. However, the knock-in level in the recent US case was considerably above the face value. First, when the metal value had been considerably above the face value for an extended period of time, the US Mint stepped in and made the melting and exporting of coins illegal. There was a period of time when there actually was an arbitrage opportunity. One could ask why the US mint waited so long before stepping in on this issue. If they knew the option theory behind their coins, they would understand that selling their coins at face value would cause arbitrage opportunities when the option element went from deep-in-the-money to at-the-money. Would they have stepped in and knocked in their call option earlier if they had been fully aware of the embedded options in physical money?

3 Conclusion

All physical monies with face values contain embedded options. By being aware of these options, more precise valuations of coins and paper money can be made. Some of the most commonly circulating coins in the USA (and also in the UK) have gone from having their embedded option deep-in-the-money to at-the-money, then to deep-out-of-the-money, and then returning to deep-in-the-money. The value of the optionality has been high. Because the central banks and commercial banks are selling such coins at a fixed price, equal to the face value, the price and face value of such coins will only be the same when the option element is deep-in-the-money. When the option element moves towards at-the-money or out-of-the-money, arbitrage opportunities arise. Since governments are not adjusting the face values, they have to reduce the value of the coin — for example, by stopping the circulation of such coins by other means or by taking legal action against the exporting and melting of coins.

A better understanding of the option element in coins is of particular importance for central banks. Based on option theory, it looks like central banks should potentially step in earlier to avoid arbitrage opportunities. If the US Mint and other central banks had been fully aware of the embedded option they were issuing in their physical money, they may have acted differently in the past. Perhaps they would not have let such arbitrage opportunities last as long as they have?

When it comes to gold and silver coins, the embedded perpetual put options are typically far out-of-the-money. Here, the standard from the governments issuing such coins is to sell them at a price equal to their real value and not at their face-value (see final version in Wilmott Magazine).

Understanding the option element in physical money gives new perspectives on how people and central banks behave, and possibly should behave, in relation to money. As more and more currency moves from physical to electronic representation, these valuations may assume increasing importance.

Appendix A

This appendix is a re-print of the statement given by the United States Mint to limit melting and exporting of certain circulating coins.

December 14, 2006
United States Mint Moves to Limit Exportation & Melting of Coins

Interim Rule Goes Into Effect Immediately

WASHINGTON — The United States Mint has implemented regulations to limit the exportation, melting, or treatment of one-cent (penny) and 5-cent (nickel) United States coins, to safeguard against a potential shortage of these coins in circulation. The United States Mint is soliciting public comment on the interim rule, which is being published in the Federal Register.

Prevailing prices of copper, nickel and zinc have caused the production costs of pennies and nickels to significantly exceed their respective face values. The United States Mint also has received a steady flow of inquiries from the public over the past several months concerning the metal value of these coins and whether it is legal to melt them.

"We are taking this action because the Nation needs its coinage for commerce," said Director Ed Moy. "We don't want to see our pennies and nickels melted down so a few individuals can take advantage of the American taxpayer. Replacing these coins would be an enormous cost to taxpayers."

Specifically, the new regulations prohibit, with certain exceptions, the melting or treatment of all one-cent and 5-cent coins. The regulations also prohibit the unlicensed exportation of these coins, except that travelers may take up to $ 5 in these coins out of the country, and individuals may ship up to $ 100 in these coins out of the country in any one shipment for legitimate coinage and numismatic purposes. In all essential respects, these regulations are patterned after the Department of the Treasury's regulations prohibiting the exportation, melting, or treatment of silver coins between 1967 and 1969, and the regulations prohibiting the exportation, melting, or treatment of one-cent coins between 1974 and 1978.

The new regulations authorize a fine of not more than $ 10,000, or imprisonment of not more than five years, or both, against a person who knowingly violates the regulations. In addition, by law, any coins exported, melted, or treated in violation of the regulation shall be forfeited to the United States Government.

The regulations are being issued in the form of an interim rule, to be effective for a period of 120 days from the time of publication. The interim rule states that during a 30-day period from the date of publication, the public can submit written comments to the United States Mint on the regulations. Upon consideration of such comments, the Director of the United States Mint would then issue the final rule.

Those interested in providing comments to the United States Mint regarding this interim rule must submit them in writing to the Office of Chief Counsel, United States Mint, 801 9th Street, N.W., Washington D.C. 20220, by January 14, 2007. The interim rule appears on the United States Mint website at www.usmint.gov. The United States Mint will make public all comments it receives regarding this interim rule, and may not consider confidential any information contained in comments.

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