Espen Gaarder Haug

A Look in the Antimatter Mirror

The Collector, **Espen Gaarder Haug***

Abstract

Take a look at yourself in the mirror. You will hopefully see a reasonably symmetric image of the real you, though your left and right sides have reversed. In this article I will explore an amazing antimatter-mirror that I recently tumbled over in the files of quantitative finance.

Garbage in, garbage out?

One day when the market was very quiet, I got bored. As an option trader (and formula collector) what else could I do but play around with my Black-Scholes-Merton calculator (a computer implementation of the famous Black and Scholes (1973)/Merton (1973) Nobel prize formula[1]? Just for fun, I wondered what would happen if I dared to input negative volatility in the formula. My state-of-the-art self-made options system returned a negative number; at least I didn't get a "catastrophic error: restart immediately" message. Options cannot take negative values, so I guess garbage in, garbage out.

But wait, could there be something to it? By playing around with it a bit more I found, quite interestingly, that inputting negative volatility for a call option, then multiplying the result by negative one actually gave me the value of a put, and vice versa. Some time ago I had listened to Nobel prize winner Richard Feynman lecture on "Einstein's Relativity, Symmetry, and Space-Time", see Feynman (1997). The talk on symmetry, matter and antimatter had been particularly fascinating. Had I just looked into a financial parallel to matter and antimatter? Before we move on, allow me to shortly introduce you to the history behind the discovery of antimatter.

Does antimatter matter?

In 1889, the physicist Arthur Schuster wrote a letter to the journal "Nature", essentially stating the question "If there is negative electricity, then why not negative gold as yellow as our own?" For thirty years, Schuster's letter gathered dust. In 1929 the English physicist Paul[2] Dirac wrote down a equation predicting the existence of antiparticles, underlining the duality idea of Arthur Schuster. Soon scientists set out on an adventure in search of antiparticles. In 1932 Carl Anderson discovered the antielectron (known as a positron). By 1956 the existence of antiparticles was well established. Today physicists speak of protons and antiprotons, neutrino and antineutrino, quarks and antiquarks . . . you name it, it has its opposite. As predicted by Dirac, it seems that every particle has an antiparticle with exactly the same mass but opposite charge. The world of antimatter can best be described as a mirror image of our own world, see Fraser (2000). If a particle and its antiparticle meet, they annihilate each other and liberate their entire mass in the form of energy. Annihilation is the only phenomenon that is fully efficient in converting mass to energy in accordance with Einstein's law, $E = mc^2$. When a nuclear bomb explodes, for instance, only a fraction of 1 percent of the atomic mass is converted into energy. When matter and antimatter collide, 100 percent of the mass disappears (turns into energy), see Lederman (1993) page 210.

Leading physicists claim that antimatter can be very useful in the future, for such endeavors as generating super clean, super efficient energy.

Let's fantasize for a while about how such an antimatter world appear in mathematical finance. If Schuster could suggest negative gold, then why not negative stock prices and strike prices? In such an antimatter world, the volatility would probably also be negative. And then to have a complete theory, there would have to be a connection between options in our world and in the antimatter mirror world.

Back to the trading floor

While playing with negative volatility in my Black-Scholes implementation I got very excited . . . a new rule of the universe?[3] Almost immediately I contacted the Wizard of option symmetries and magic tricks, Dr. Peter Carr[4], currently at NYU. Almost immediately he replied that this

*Thanks to Alexander (Sasha) Adamchuk, Peter Carr, Erik Stettler and Gordon Fraser for many useful comments on this paper. The antimatter Collector is naturally responsible for any remaining errors.

finding was very interesting, and closely related to a recent working paper by Peskir and Shiryaev (2000). Using sophisticated mathematics Peskir and Shiryaev (2000) demonstrates what they named put-call duality:

$$c(S, X, T, r, b, \sigma) = p(-S, -X, T, r, b, -\sigma)$$

where c is the Black-Scholes-Merton call option formula, p is a put, S is the asset price, X is the strike price, T is time to maturity, r is the risk-free rate, b is the cost of carry for the underlying security, and σ is the volatility of the asset. It was actually the genius Alexander (Sasha) Adamchuk from the University of Chicago who first introduced Shiryaev and some other people to the symmetry just mentioned, what he had coined supersymmetry (at the conference on Quantitative Risk Management in Finance, July 31 — August 5, 2000, Carnegie Mellon University, Pittsburgh[5]). Without even thinking about it, I had just used my option calculator to combine the Adamchuk/Peskir-Shirayev antimatter-duality/supersymmetry with a simple state-space transformation:

$$k \times c(S, X, T, r, b, \sigma) = c(k \times S, k \times X, T, r, b, \sigma)$$

where k is a constant. This leads to the interesting antimatter-option-mirror that I had just been looking into (this is what Sasha had coined supersymmetry)

$$c(S, X, T, r, b, \sigma) = -p(S, X, T, r, b, -\sigma)$$

and naturally

$$p(S, X, T, r, b, \sigma) = -c(S, X, T, r, b, -\sigma)$$

But why is it not perfectly symmetrical? A call does not equal an antimatter-call, and a put does not equal an antimatter-put. Interestingly this is also what we see in the matter and antimatter world, in Feynman's own words[6]:

> "If we made a left-hand clock, but made it out of the other kind of matter, antimatter instead of matter, it would run in the same way (as a right hand matter clock). So what has happened is instead of having two independent rules in our list of symmetries, two of these rules goes together and make a new rule, which says that to the right is symmetrical with antimatter to the left. "

In the option world we get the similar rule

> A matter-call (right-handed derivative instrument) is symmetrical with an antimatter-put (left-handed derivative instrument), and vice versa.

The product really stems from the fact that the results for both puts and calls can be obtained if one introduces a polarity variable, for example either $+1$ or -1 The polarity variables enter through the boundary conditions and are thus present in the solution. At least this is part of the story.

Not only does this result hold for the option value, but naturally also for all the option Greeks: delta, theta, rho etc. For example the delta of a call will equal the antimatter-delta of a put.

The matter/antimatter-option rule is not only stimulating for the imaginary brain cells, but the result is actually very robust and simplifies coding and implementation of many derivatives models. You need only the formula and code for a call or for a put, as one can easily be translated into the other using an antimatter-mirror. As Dr. Carr the symmetry Wizard pointed out to me, the result only holds when the pricing formulas involve σ times $\sqrt{T}$, rather than $\sqrt{\sigma^2 T}$. The result can easily be extended to several exotic options, like barriers and Asian options.

In the financial markets, empirical research suggests that most assets and commodities have

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leptokurtic distributions (fat tails and peaked top). We can use option models to describe the distribution by using higher order moments like skewness and kurtosis[7]. In our matter world the third moment, skewness, can be estimated as

$$\text{Skewness} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^3 / n}{\sigma^3} \tag{1}$$

where x_i is the asset returns, $\bar{x}$ is the mean of the asset return, n is the number of observations, and σ is the second moment of the distribution (the volatility). Since volatility is negative in the antimatter world, skewness will change sign.

The fourth moment of a distribution known as Kurtosis (Pearson) is defined as

$$\text{Kurtosis} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4 / n}{\sigma^4} \tag{2}$$

From this we can see that the antimatter kurtosis would still be positive in both the matter and antimatter world.

Figure 1 shows two distributions generated by a very flexible option valuation model that also can take into account a wide specter of distributions, more precisely, the Rubinstein (1998) extension of the Cox, Ross, and Rubinstein (1979)/Rendleman and Bartter (1979) binomial model. This is essentially a discrete implementation of the Black-Scholes-Merton model extended for distributions different from log-normal using an Edgeworth expansion (basically a discrete implementation of the Jarrow and Rudd (1982) formula).

The distribution to the right is generated by using normal input parameters (the matter world), and the probability distribution to the left was generated on the basis of antimatter input, that is negative stock price, negative volatility and opposite sign skewness. The stock price process in the antimatter-binomial tree is negative, and the distribution is the antimatter-mirror image of the distribution generated by the normal input parameters. Remember one of the rules in particle physics is that particles and antiparticles have the exact same mass, but opposite charges. Similarly the two distributions have exactly the same probability mass, but different charges. But just as in a mirror (and as in the matter and antimatter world.) a matter-call equals an antimatter-put and vice versa. More formally, when including skewness or kurtosis in our valuation we get the following matter antimatter relationship

$$\begin{aligned} c(S, X, T, r, b, \sigma, \kappa, \xi) \\ = -p(S, X, T, r, b, -\sigma, -\kappa, \xi) \end{aligned}$$

and naturally also

$$p(S, X, T, r, b, \sigma, \kappa, \xi) = -c(S, X, T, r, b, -\sigma, -\kappa, \xi)$$

where κ is the skewness and ξ is the kurtosis of the matter distribution. This result holds for Edgeworth and Gram-Charlier[8] expansions of the Black-Scholes-Merton formula, as well as for multiple discrete implementations. As we have demonstrated, this is a very general result.

Option-antimatter Philosophy

Yes I know you already are asking: has the Collector turned nuts? My imagination has certainly gone wild, and I am certainly approaching a field where my knowledge is approaching zero (if not negative). But before you judge me let me offer a quote from the Master of the Universe

> "I am enough of an artist to draw freely upon my imagination. Imagination is more important than knowledge. Knowledge is limited."
>
> Albert Einstein

Imagination is certainly not only important in astrophysics. Long Term Capital Management also believed themselves to be the Masters of the Universe, with multiple Nobel prize winners and knowledge that others could only dream about, but still their complete lack of market imagination made LTCM blow up in a Big Bang, leaving behind only a black hole.

Please hang on because it now gets even more interesting. I now ask whether we in an antimatter world could possibly also replace positive probabilities with negative ones[9]. Negative probabilities would twist the probability mass around the x axis and give negative option values (not to be confused with shorting an option). At first this would seem crazy, but on the other hand, this would make our matter and antimatter options "equivalent" to the particle world. Still we are in a

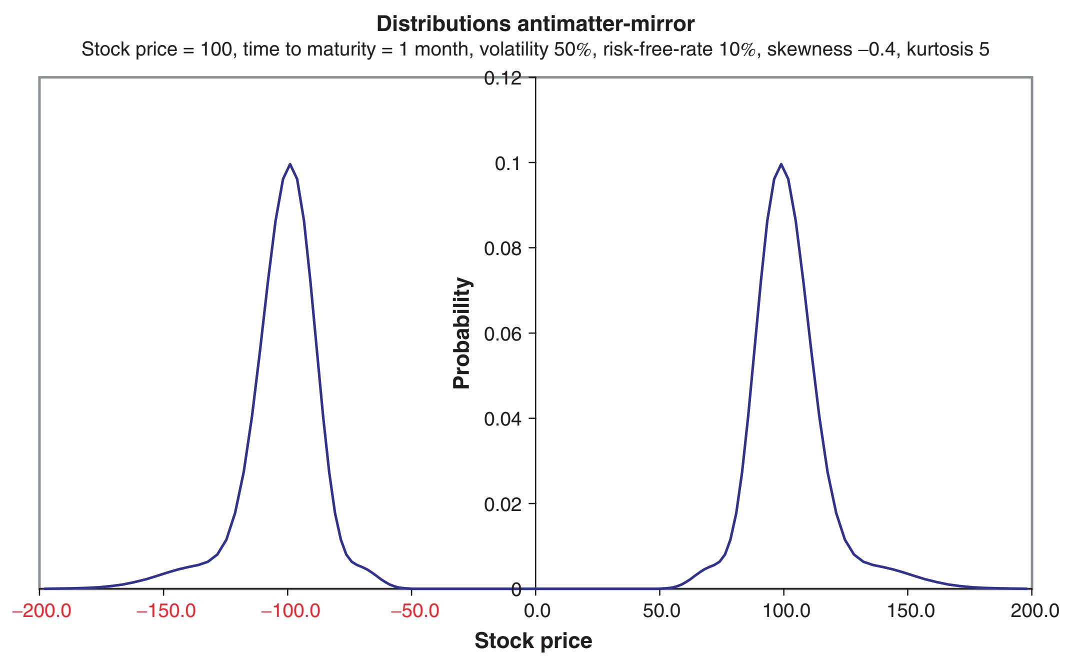

Fig. 1

"I am enough of an artist to draw freely upon my imagination. Imagination is more important than knowledge. Knowledge is limited."

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left-right matter antimatter world, in other words we have a lack of perfect symmetry.

Matter and antimatter are not perfectly symmetric as one first assumed (a left-handed neutrino does not equal a left-hand antineutrino but rather a right-hand antineutrino[10]. It was not long before physicists all asked themselves (and still are asking) if there is something more basic underneath it all. Is there a God particle[11] that really is perfectly symmetrical? In the search for perfect symmetry one physicist proposed the super sting theory. Could it be that the "shapes" of the particles one observes - quarks, antiquarks etc. not are different particles, but rather super strings (God particles) that vibrate with the speed of light in different modes? Could different vibrations generates different particles just as different vibrations in a guitar string generate an almost infinite number of melodies?

But wait a minute. Don't we have a closed parallel in finance: the God particle of Brownian motion? Brownian motion leads to the bell shaped normal distribution which is perfect symmetric. Thus a call equals an antimatter-call and a put equals an antimatter-put[12]. From Brownian motion we can generate almost an infinite number of stochastic processes and distributions. Now going from mathematical finance back to physics, could it be that the God particles are simply super strings vibrating/spinning around with Brownian motion? Then by combining such God particles one could generate almost any particle-distribution-shape that again could explain the different particles we observe?

Conclusion

Back to reality. When it comes to option pricing, garbage-in (negative stock price, strike and volatility) does not necessary lead to garbage-out, but rather a very interesting relationship regarding matter and antimatter. These results are not only extremely fascinating but also have very useful practical applications. For many option models there is essentially no need for developing and implementing separate models for call and put options; all one needs is a formula for a call or a put, then by inputting the antimatter equivalent stock price, strike price, volatility and higher order moments we will get the value for a put and vice versa.

More important, if a matter and an antimatter-option met each other they would annihilate each other. Instead of releasing a huge amount of energy as in the particle world, this would possibly release a lot of money. But who would get all that money . . . God himself? Or would it all simply disappear, as this probably only could happen when a matter and antimatter universe collides (through a black hole?), thus annihilating each other in a Big Bang?

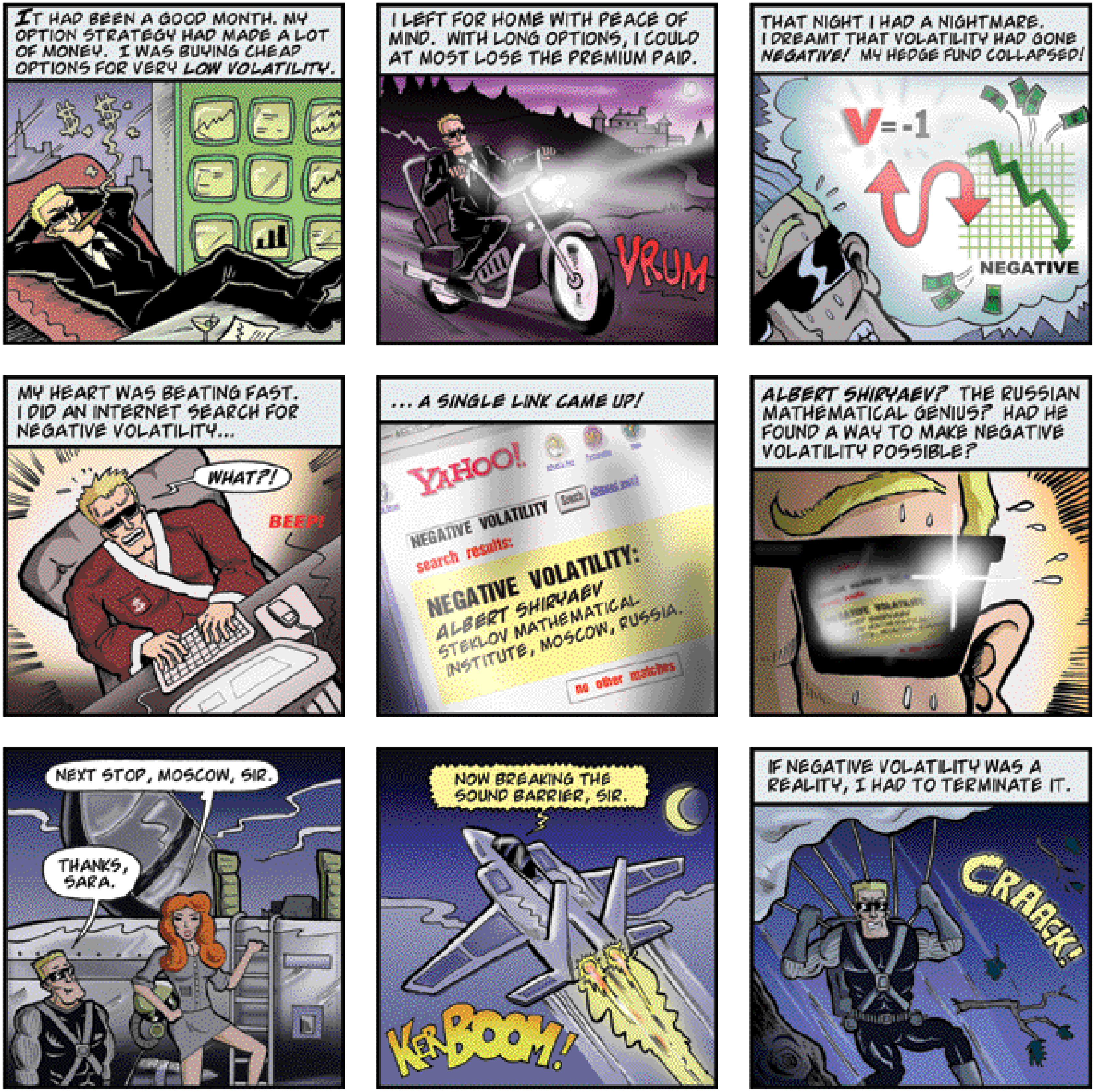
FOOTNOTES

1. See Haug (1997) for implementation of this formula and many others.
2. Not to be confused with Paul Wilmott who was not yet born at the time.
3. A rule not even described in the two volume Bible of quantitative finance: "Dr. Paul on Quantitative Finance", see Wilmott (2000).
4. Dr. Carr was one of the first to demonstrate the use of what is known as put-call symmetry, Carr and Bowie (1994). He later published a whole series of papers demonstrating the use of different symmetry principles in finance (Carr and Chou 1997, Carr and Chou 1998, Carr, Ellis, and Gupta 1998). Bates (1991) was the first to publish this form of put-call symmetry. See also (Haug 1998, Haug 2001a, Haug 2001b) for several implications of reflection and barrier symmetries.
5. Adamchuk's most recent talk on this topic was at the Kolmogorov Memorial Readings: "From Supernova to Discovery of Supersymmetry in Finance. New Vistas in Mathematical Foundations of Finance", The University of Chicago, April 2001: http://finmath.com/Chicago/NAFTCORP/Kolmogorov_Memorial_Readings_2001.html. By the way Adamchuk would not accept that I had coined him genius, that he is a genius is possibly not true? but how can one deny the reality, "You must be able to distinguish between what is true and what is real" Albert Einstein. Still not convinced, take a look at http://finmath.com/ Chicago/NAFTCORP/Sasha.html, a picture tells more than thousand words.
6. Listen to Feynman (1997) audio cd.
7. One of the first papers describing this idea was Jarrow and Rudd (1982).
8. See Corrado and Su (1996) and Brown and Robinson (2001).
9. I am not the first scientist to introduce negative probabilities, see Feynman (1987). However I am probably the first mad quasi scientist introducing this in the fields of finance.
10. In physics we are not referring to symmetry in geometrical figures but rather to the symmetries of actions. For a good reference on symmetries in particle physics and such deep concept as the CPT Theorem see Lee (1988).
11. Read Lederman (1993) for the story of the search for the God particle.
12. The first to suggest using a normal distribution for option valuation was probably Bachelier (1900).

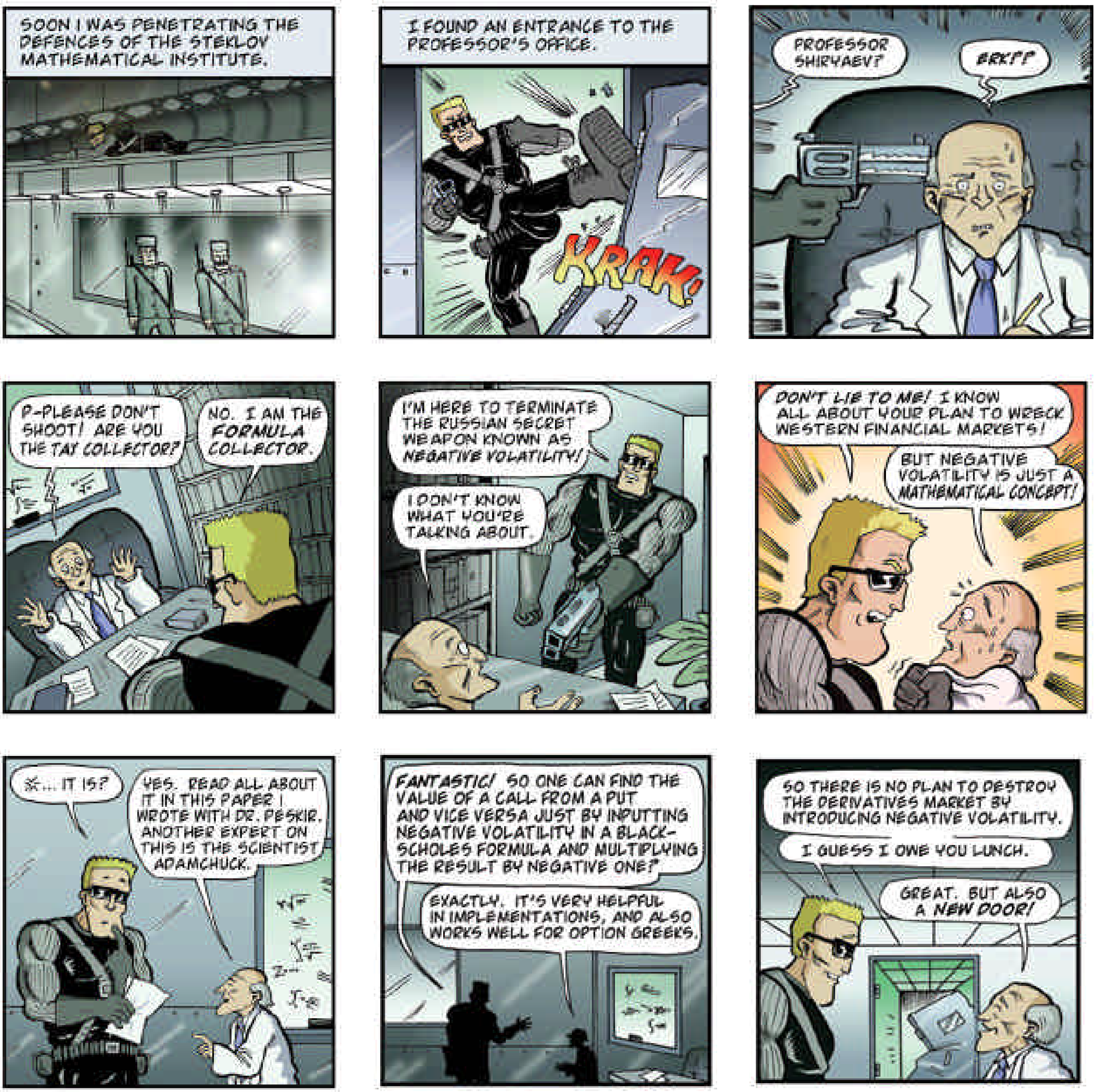
REFERENCES

■ Adamchuk, A. (1998): "From Supernova to Discovery of Supersymmetry in Finance", *New Vistas in Mathematical Foundations of Finance. unpublished, University of Chicago*.
■ Bachelier, L. (1900): *Theory of speculation in: P. Cootner, ed., 1964, The random character of stock market prices*, MIT Press, Cambridge, Mass.
■ Bates, D. S. (1991): "The Crash of '87: Was It Expected? The Evidence from Options Markets, " *Journal of Finance*, 46(3), 1009–1044.
■ Black, F., and M. Scholes (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637–654.
■ Brown, C., and D. Robinson (2001): "Skewness and Kurtosis Implied by Option Prices: A Correction," *Journal of Financial Research*, Forthcoming.
■ Carr, P., and J. Bowie (1994): "Static Simplicity," *Risk Magazine*, 7(8).
■ Carr, P., and A. Chou (1997): "Breaking Barriers," *Risk Magazine*, pp. 139–145.
■ ______ (1998): "Static Hedging of Complex Barrier Options," *Banc of America Securities Working Paper, http://www.math.nyu.edu/research/carrp/papers/*.
■ Carr, P., K. Ellis, and V. Gupta (1998): "Static Hedging of Exotic Options," *Journal of Finance*, 53.
■ Corrado, C. J., and T. Su (1996): "Skewness and Kurtosis in S&P 500 Index Returns Implied by Option Prices," *Journal of Financial Research*, XIX, 175–192.
■ Cox, J. C., S. A. Ross, and M. Rubinstein (1979): "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, 7, 229–263.
■ Feynman, R. P. (1987): "Negative Probability," *First published in the book Quantum Implications : Essays in Honour of David Bohm, by F. David Peat (Editor), Basil Hiley (Editor) Routledge & Kegan Paul Ltd, London & New York*, pp. 235–248.
■ ______ (1997): *Six Not-So-Easy Pices, Lecture 2: Symmetry in Physical Laws, May 20, 1963*. Addison Wesley.
■ Fraser, G. (2000): *Antimatter, the Ultimate Mirror*. Cambridge University Press.
■ Haug, E. G. (1997): *The Complete Guide To Option Pricing Formulas*. McGraw-Hill, New York.
■ ______ (1998): "Put-Call Barrier Transformations," *Working Paper Tempus Financial Engeneering, http://home.online.no/~espehaug/*.
■ ______ (2001a): "Closed Form Valuation of American Barrier Options," *International Journal of Theoretical and Applied Finance*, XIX, 175–192.
■ ______ (2001b): "First-then-knockout options," *Wilmott*, August.
■ Jarrow, R., and A. Rudd (1982): "Approximate Option Valuation for Arbitrary Stochastic Processes," *Journal of Financial Economics*, 10, 347–369.
■ Lederman, L. (1993): *The God Particle*. Dell Publishing.
■ Lee, T. D. (1988): *Symmetries, Asymmetries, and the World of Particles (Jesse and John Danz Lecture Series)*. University of Washington Press Seattle & London.
■ Merton, R. C. (1973): "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4, 141–183.
■ Peskir, G., and A. N. Shiryaev (2000): "A note on the Put-Call Parity and a Put-Call Duality," *Working Paper Series No. 78., CAF, University of Aarhus, http://www.cls.dk/caf/wp/wp.html*.
■ Rendleman, R. J., and B. J. Bartter (1979): "Two-State Option Pricing," *Journal of Finance*, 34, 1093–1110.
■ Rubinstein, M. (1998): "Edgeworth Binomial Trees," *Journal of Derivatives*, XIX, 20–27.
■ Wilmott, P. (2000): *Paul Wilmott on Quantiative Finance*. Wiley.

THE COLLECTOR IN FROM RUSSIA WITH VOL



Cartoon: Sebastian Conley **Words:** Espen Gaarder Haug


SOON I WAS PENETRATING THE DEFENCES OF THE STEKLOV MATHEMATICAL INSTITUTE.
I FOUND AN ENTRANCE TO THE PROFESSOR'S OFFICE.
KRAK!
PROFESSOR SHIRYAEV?
ERK!?!
P-PLEASE DON'T SHOOT! ARE YOU THE TAX COLLECTOR?
NO. I AM THE FORMULA COLLECTOR.
I'M HERE TO TERMINATE THE RUSSIAN SECRET WEAPON KNOWN AS NEGATIVE VOLATILITY!
I DON'T KNOW WHAT YOU'RE TALKING ABOUT.
DON'T LIE TO ME! I KNOW ALL ABOUT YOUR PLAN TO WRECK WESTERN FINANCIAL MARKETS!
BUT NEGATIVE VOLATILITY IS JUST A MATHEMATICAL CONCEPT!
*... IT IS?
YES. READ ALL ABOUT IT IN THIS PAPER I WROTE WITH DR. PESKIR. ANOTHER EXPERT ON THIS IS THE SCIENTIST ADAMCHUCK.
FANTASTIC! SO ONE CAN FIND THE VALUE OF A CALL FROM A PUT AND VICE VERSA JUST BY INPUTTING NEGATIVE VOLATILITY IN A BLACK-SCHOLES FORMULA AND MULTIPLYING THE RESULT BY NEGATIVE ONE?
EXACTLY. IT'S VERY HELPFUL IN IMPLEMENTATIONS, AND ALSO WORKS WELL FOR OPTION GREEKS.
SO THERE IS NO PLAN TO DESTROY THE DERIVATIVES MARKET BY INTRODUCING NEGATIVE VOLATILITY.
I GUESS I OWE YOU LUNCH.
GREAT. BUT ALSO A NEW DOOR!